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CPT violating decoherence and LSND: a possible window to planck scale physics

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Abstract: Decoherence has the potential to explain all existing neutrino data including LSND results, without enlarging the neutrino sector. This particular form of CPT violation can preserve the equality of masses and mixing angles between particle and antiparticle sectors, and still provide seizable differences in the oscillation patterns. A simplified minimal model of decoherence can explain the existing neutrino data as well as the standard three oscillation scenario, while making dramatic predictions for the upcoming experiments. Such a model can easily accomodate the LSND result but cannot fit the spectral distortions seen by KamLAND. Some comments on the order of the decoherence parameters in connection with theoretically expected values from some models of quantum-gravity are given. In particular, the quantum gravity decoherence as a primary origin of the neutrino mass differences scenario is explored, and even a speculative link between the neutrino mass-difference scale and the dark energy density component of the Universe today is drawn.

Keywords: Discrete and Finite Symmetries, Solar and Atmospheric Neutrinos, Neutrino Physics.

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1. Introduction: the LSND puzzle and abandoning of symmetries

Although the Special Theory of Relativity, a theory of flat space time physics based on Lorentz symmetry, is very well tested, and in fact next year it celebrates a century of enormous success, having passed very stringent experimental precision tests, a quantum theory of Gravity, that is a consistent quantized version of Einstein's General Relativity, a dynamical model for curved space times, still eludes us. The main reason for this is the lack of any concrete observational evidence on the structure of space time at the characteristic scale of quantum gravity (QG), the (four-dimensional) Planck mass scale $M_P = 10^{19}$ GeV. The situation concerning QG is to be contrasted with that characterizing its classical counterpart, General Relativity, and the quantum version of Special Relativistic field theories, both of which can be tested to enormous precision to date. For instance, the perihelion precession of planets such as mercury, as well as the deflection of light by the sun, made the General Theory of Relativity an instant success, almost immediately after it was proposed in 1915. Similarly, the modern versions of relativistic quantum field theories, including the foundations of the standard model of particle physics, which have been tested extremely well up to now, leave little doubt on the validity of Special Relativity as a quantum theory of flat space time, that is in the absence of gravitational effects.

One would hope that, like any other successful physical theory, a physically relevant quantum theory of gravity should lead to experimental predictions that should be testable in the foreseeable future. In the QG case, however, such predictions may be subtle, due to the extremely weak nature of the gravitational interaction as compared with the rest of the known interactions in nature. Indeed, any hope for a true "phenomenology of QG" may be easily proven to be wishful thinking, if all the symmetries and properties that characterize the current version of what is called particle physics phenomenology are still valid in a full quantum theory of gravity. This is mainly due to the fact that the dimension-full coupling constant of gravity, the Newton constant $G_N = 1/M_P^2$, appears as a very strong suppression factor of any physical observable that could be associated with predictions of quantum gravity, in a theory in which Lorentz invariance and the laws of quantum mechanics (or better quantum field theory), such as unitary temporal evolution and locality of interactions, still hold.

In recent years, however, more and more physicists contemplate the idea that such laws, which are characteristic of a flat space time quantum field theory, may not be valid in a full theory of curved space times. For instance, the CPT theorem, one of the most profound results of quantum field theory [1], which is a consequence of Lorentz invariance, locality, as well as quantum mechanics (specifically unitary evolution of a system), may not characterize a quantum gravity theory. The possibility of a violation of CPT invariance by quantum gravity has been raised in a number of theoretical models of QG that go beyond conventional local quantum field theoretic treatments of gravity [2, 4, 5, 6].

In a phenomenological, rather model-independent setting, CPT Violation has been invoked recently [8] in an attempt to explain within a three generation scenario, without the introduction of sterile neutrinos, a puzzling experimental result in neutrino physics, namely the claims from LSND Collaboration [7] on evidence of oscillations in the antineutrino sector $\overline{\nu}_\mu \to \overline{\nu}_e$, due to observed excess of $\overline{\nu}_e$ events, but lack of evidence (depending though on interpretation) for oscillations in the corresponding neutrino sector $\nu_\mu \to \nu_e$. In order to explain these results within the conventional oscillation scenario, one should invoke different neutrino mass differences in the particle and antiparticle sectors of the model. This would signify CPT violation. Specifically, from solar and atmospheric neutrino data, the mass squared differences that account for the observed oscillations are of order $\Delta m^2 \sim 10^{-5}\,\mathrm{eV}^2$ and $10^{-3}\,\mathrm{eV}^2$, respectively, while the LSND result would require $\Delta m^2 \sim 10^{-1}\,\mathrm{eV}^2$, which would explain the suppressed signal in the neutrino sector and the strong signal in the antineutrino one. To reconcile, therefore, the LSND result with the rest of the available neutrino data, in conventional oscillation scenaria, one would have to invoke CPT violating mass differences between neutrinos and antineutrinos of order $|m_{\overline{\nu}}^2 - m_\nu^2| \sim 10^{-1}\,\mathrm{eV}^2$. Although such a CPT violation may look rather drastic, the most stringent limit we have on the CPT symmetry today, the one coming from the neutral kaon system, if reinterpreted as a limit on the possible differences in mass squared, gives: $|m^2(K^0) - m^2(\bar{K}^0)| < 0.25\,\mathrm{eV}^2$ showing that such differences are just barely explored even in the kaon system.

Subsequent global analyses of neutrino data [9], however, seem to disfavor two and three generation CPT violating scenaria [8, 6] thereby leaving the CPT violating scenaria involving sterile neutrinos as the only surviving possibility for the explanation of the LSND results. Specifically, it has been argued in [10], that allowing for CPT violation, the mixing matrix elements with the forth generation between neutrino and antineutrino sectors need no longer be the same, thereby evading stringent experimental constraints that killed three-generation models.

Is, however, CPT violation evidenced only as an inequality between neutrino-antineutrino masses the only way a violation of this symmetry can manifest itself in nature?

Such a question becomes extremely relevant for the case of LSND, because it is possible that other mechanisms leading to CPT violation exist, unrelated, in principle, to mass differences between particles and antiparticles. Such additional mechanisms for CPT violation may well be capable of explaining the LSND results within a three generation scenario without invoking a sterile neutrino (a scenario which, on the other hand, is getting totally excluded as new experimental data become available). It is therefore necessary to explore whether alternative ways exist to account for the LSND result without invoking extra (sterile) neutrino states.

As we shall argue in this article, quantum decoherence may be the key to answer this question. Indeed, quantum decoherence in matter propagation occurs when the matter subsystem interacts with an 'environment', according to the rules of open-system quantum mechanics. At a fundamental level, such a decoherence may be the result of propagation of matter in quantum gravity space-time backgrounds with 'fuzzy' properties, which may be responsible for violation of CPT in a way not necessarily related to mass differences between particles and antiparticles.

It is the point of this work to claim that decoherence scenaria combined with conventional oscillations, led as usual by the mass differences between the mass eigenstates, can account for all the available neutrino data, including the LSND result, within a three generation model. The LSND result would then evidence CPT violation in the sense of different decohering interactions between particle and antiparticle sectors, while the mass differences between the two sectors remain the same. For detailed reviews of decoherence in two-level systems, with emphasis on phenomenology including neutrinos, we refer the reader to the literature [11]. In what follows we shall only expose the very basic features, which we shall make use of in this work in order to perform our phenomenological analysis. It is important to stress that in decoherence scenaria CPT symmetry is violated in its strong form, in the sense of the CPT operator not being well defined. This is an important issue which we now come to discuss briefly.

2. Quantum gravity and decoherence: brief review

A characteristic example of such a violation occurs in quantum gravity models that involve singular space-time configurations, integrated over in a path integral formalism, which are such that the axioms of quantum field theory, as well as conventional quantum mechanical behavior, cannot be maintained [3]. Such configurations consist of wormholes, microscopic (Planck size) black holes, and other topologically non-trivial solitonic objects, such as *geons* [12], *etc.* Collectively, we may call such configurations *space time foam*, a terminology given by J.A. Wheeler [13] who first conceived the idea that the structure of the *quantum* space time at Planck scales (10^{-35} m) may indeed be fuzzy.

It has been argued that, as result, a mixed state description must be used (*QG-induced decoherence*) [3, 2], given that such objects cannot be accessible to low-energy observers, and as such must be traced over in an effective field theory context. For the case of microscopic black holes this can be readily understood by the loss of information across microscopic event horizons, as a corollary of which it has been argued [14], already back in

1978, that CPT invariance in its strong form must be abandoned in a foamy quantum gravity theory. Such a breakdown of CPT symmetry is a fundamental one, and, in particular, implies that a proper CPT operator may be *ill defined* in such QG decoherence cases.

The main reason behind such a failure is the non factorisability of the super-scattering matrix, \$, connecting asymptotic "in" and "out" density matrices [3]. The latter describe *mixed* quantum states due to the existence of a quantum-gravitational "environment" of microscopic singular space-time configurations. The presence of the environment leads to *decoherence*, like in any other case of an open quantum-mechanical system, implying an evolution from an initial pure quantum mechanical state $|\phi >$ to a mixed state $\rho_\psi = \mathrm{Tr}|\psi >< \psi|$ (the trace is over unobserved degrees of freedom, 'lost' in the environment). This in turn implies [14] the *non invertibility* of the \$-matrix, as a quantum mechanical operator acting on density matrices. The associated CPT non invariance is then linked to the fact that if CPT were a well defined operator, stemming from the invariance of the system under its action, then \$ would have been invertible, in obvious contradiction with the information loss and decoherence inherent to the problem.

As emphasized recently in [15] this would also imply that the concept of *antiparticle*, may be *ill defined*, with interesting testable consequences in multiparticle situations, such as those encountered in (EPR-like entangled) meson factories. Such CPT violation is distinct from other violations of this symmetry as a result of Lorentz symmetry breakdown in the so-called Standard Model extension (SME) [5], or locality violations [6], which have been considered in the recent literature as other experimentally testable approaches to quantum gravity. It should be noted that these latter violations of symmetries may or may not coexist with QG decoherence, given that Lorentz invariance is not necessarily inconsistent with decoherence [16].

Some caution should be paid regarding CPT Violation through decoherence. As emphasized in [14], from a formal view point, the non-invertibility of the \$-matrix, which implies a strong violation of CPT in the sense described above, does not preclude a softer form of CPT invariance, in the sense that any strong form of CPT violation does not necessarily have to show up in any single experimental measurement. This implies that, despite the general evolution of pure to mixed states, it may still be possible in the laboratory to ensure that the system evolves from an initial pure state ψ to a single final state ϕ, and that the weak form of CPT invariance is manifested through the equality of probabilities between the states ψ, ϕ:

$$P(\psi \to \phi) = P(\theta^{-1}\phi \to \theta\psi)\,, \tag{2.1}$$

with θ a CPT operator acting on the subspace of pure state vectors in the laboratory, such that for density matrices $\Theta\rho = \theta\rho\theta^\dagger$, $\theta^\dagger = -\theta^{-1}$, and $\$^\dagger = \Theta^{-1}\$\Theta^{-1}$, but $\$^{\dagger} \ne \$^{-1}$, unless full CPT invariance holds, which is the case of no decoherence. If this is the case, then the decoherence-induced CPT violation will not show up in any experimental measurement. It is therefore important to check whether it is possible to test experimentally which case is realized in nature. Fortunately this can be done in a clear way.

Indeed, the main consequence of the quantum-gravity induced decoherence is the *modified* non-quantum mechanical evolution of a matter system under consideration, as a result of the interaction with the QG environment [2]. Such an evolution will result in many observable consequences, which may be testable in the near future with a precision that in some cases can reach Planck scale sensitivity. Historically, the first probe of such non-quantum mechanical decoherence and CPT violating behavior (in the sense of the strong form violation of [14]) is the neutral kaon system [2, 4], in both single-particle experiments, such as the CPLEAR [17] and KTeV experiments [18], as well as multi-particle situations, such as kaon (ϕ) factories [19]. Other neutral mesons, such as B mesons [20], can also be used as sensitive probes of QG decoherence.

In single-particle neutral meson experiments one looks at time profiles of asymmetries [4]. The mass difference between the energy eigenstates K_L, K_S is responsible for the 'bulk' behavior of the asymmetry, for instance leading to an oscillatory pattern in some of the asymmetries, such as $A_{2\pi}$, decoherence on the other hand can be responsible for a *slight distortion* of such a behavior [4]. The non observation of such distortions results in stringent bounds on the decoherence parameters. Mathematically, one does not have to have a detailed knowledge of the dynamics of quantum gravity in order to perform experimental low-energy tests of decoherence. This can be achieved following the so-called Lindblad or mathematical semi-groups approach to decoherence [21], which is a very efficient way of studying open systems in quantum mechanics. The time irreversibility in the evolution of such semigroups, which is linked to decoherence, is inherent in the mathematical property of the lack of an inverse in the semigroup. This approach has been followed for the study of quantum-gravity decoherence in the case of neutral kaons in [2, 4].

In the parameterization of [2] for the decoherence effects, using three decoherence parameters with dimensions of energy, α, β, γ, one finds the following bounds from the CPLEAR experiment [17]:

$$\begin{aligned} &\alpha < 4 \times 10^{-17}\,\mathrm{GeV}, |\beta| < 2.3 \times 10^{-19}\,\mathrm{GeV}\,, \\ &\gamma < 3.7 \times 10^{-21}\,\mathrm{GeV} \end{aligned} \tag{2.2}$$

where the positivity of ρ, required by the fact that its diagonal elements express probability densities, implies $\alpha, \gamma \geq 0$, and $\alpha\gamma \geq \beta^2$.

The Lindblad approach to decoherence does not require any detailed knowledge of the environment, apart from energy conservation, entropy increase and complete positivity of the (reduced) density matrix $\rho(t)$ of the subsystem under consideration. The basic evolution equation for the (reduced) density matrix of the subsystem in the Lindblad approach is *linear* in $\rho(t)$ and reads:

$$\frac{\partial \rho}{\partial t} = -i[H_{\mathrm{eff}}, \rho] + \frac{1}{2}\sum_j \left([b_j, \rho(t) b_j^\dagger] + [b_j \rho(t), b_j^\dagger]\right), \tag{2.3}$$

where H_{eff} is the effective hamiltonian of the subsystem, and the operators b_j represent the interaction with the environment, and are assumed bounded. Notice that the Lindblad part cannot be written as a commutator (of a hamiltonian function) with ρ. Environmental contributions that can be cast in hamiltonian evolution (commutator form) are absorbed in H_{eff}.

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It must be noted at this stage that the requirement of complete positivity, which essentially pertains to the positivity of the map $\rho(t)$ as the time evolves in the case of many particle situations, such as meson factories (two-kaon states (ϕ-factory), or two-B-meson states *etc.*), may not be an exact property of quantum gravity, whose interactions with the environment could be *non linear* [22]. Nevertheless, complete positivity leads to a convenient and simple parametrization, and it has been assumed so far in many phenomenological analyses of quantum gravity decoherence in generic two state systems, such as two-flavor neutrino systems [23, 24]. In fact, in the parametrization of [2], the imposition of complete positivity leads to $\alpha = \beta = 0$, leaving only $\gamma > 0$ as the only decoherence parameter in the two-state system [25].

Formally, the bounded Lindblad operators of an N-level quantum mechanical system can be expanded in a basis of matrices satisfying standard commutation relations of Lie groups. For a two-level system [2, 4] such matrices are the SU(2) generators (Pauli matrices) plus the 2×2 identity operator, while for a three level system [26], which will be relevant for our purposes in this article, the basis comprises of the eight Gell-Mann SU(3) matrices Λ_i , $i = 1, \ldots 8$ plus the 3×3 identity matrix I_{3x3}.

Let $\mathcal{J}_\mu$, $\mu = 0, \ldots 8(3)$ be a set of SU(3) (SU(2)) generators for a three(two)-level system; then, one may expand the various terms in (2.3) in terms of $\mathcal{J}_\mu$ to arrive at the generic form:

$$\begin{aligned} &\frac{\partial \rho_\mu}{\partial t} = \sum_{ij} h_i \rho_j f_{ij\mu} + \sum_{\nu} \mathcal{L}_{\mu\nu} \rho_\mu \,, \\ &\mu, \nu = 0, \ldots N^2 - 1 \,, \qquad i, j = 1, \ldots N^2 - 1 \end{aligned} \tag{2.4}$$

with $N = 3(2)$ for three(two) level systems, and f_{ijk} the structure constants of the SU(N) group. The requirement for entropy increase implies the hermiticity of the Lindblad operators b_i, as well as the fact that the matrix $\mathcal{L}$ of the the non-hamiltonian part of the evolution has the properties that $\mathcal{L}_{0\mu} = \mathcal{L}_{\mu 0} = 0$, $\mathcal{L}_{ij} = \frac{1}{2} \sum_{k,\ell,m} b_m^{(n)} b_k^{(n)} f_{imk} f_{\ell kj}$, with the notation $b_j \equiv \sum_\mu b_\mu^{(j)} \mathcal{J}_\mu$.

In the two-level case of [2] the decoherence matrix $\mathcal{L}_{\mu\nu}$ is parametrized by a 4×4 matrix, whose non vanishing entries are occupied by the three parameters with the dimensions of energy α, β, γ with the properties mentioned above. If the requirement of a completely positive map $\rho(t)$ is imposed, then the 4×4 matrix $\mathcal{L}$ becomes diagonal, with only one non vanishing entry occupied by the decoherence parameter $\gamma > 0$ [25].

3. Two generation neutrino models and decoherence: brief review

If the above formalism is applied to two-level neutrino oscillation physics, in an attempt to discuss the influence of a quantum-gravity environment on neutrino oscillations, then there are two possible forms of the matrix $\mathcal{L}$: $\mathcal{L} = \mathrm{Diag}\,(0, 0, -\gamma, 0)$ in the case where energy and lepton number are *conserved*, and $\mathcal{L} = \mathrm{Diag}\,(0, 0, 0, -\gamma)$ in the case that energy and lepton number are violated by quantum gravity, but *flavor* is conserved. In [23] the energy conserving case has been considered, with the conclusion that pure decoherence, i.e. induced oscillations in the absence of neutrino mass differences, was incompatible with the observed phenomenology of neutrino oscillations.

This is in agreement with the kaon case [4], where again the observed bulk behavior of the time profiles of the various kaon asymmetries, as well as the observed CP violation, cannot be explained solely by quantum decoherence, the latter providing only distortions to the 'bulk' behavior. Including the mass-induced oscillations, combined with decoherence, and comparing with atmospheric neutrino data, the authors of [23] managed to obtain bounds for the decoherence parameter γ in the presence of mass differences (oscillations); as in the kaon case [4], such bounds are obtained merely by the non-observation of the slight distortions to the mass-induced neutrino oscillations that the presence of decoherence would cause.

The so-obtained limits for the decoherence Lindblad parameter γ in [23] can be summarized as follows: adopting the parametrization $\gamma = \gamma_0 (E/\mathrm{GeV})^n$, they considered three cases: (a) $n = 0$, in which the experimental data imply $\gamma_0 < 10^{-23}$ GeV, (b) $n = 2$, which is the case expected in some rather optimistic (from the prospect of experimental detection) theoretical models of quantum gravity [22, 4], for which the most stringent bound derived is $\gamma_0 < 0.9 \times 10^{-27}$, and (c) $n = -1$, which is the case in which the decoherence parameter exhibits an energy dependence that mimics the conventional oscillation scenario, for which $\gamma_0 < 2 \times 10^{-21}\,\mathrm{GeV}^2$.

It is worth noticing, that as far as case (b) is concerned, at least by dimensional considerations, one is tempted to assume theoretical values of the decoherence parameter which are of the form E^2/M_{QG}, with M_{QG} the effective quantum gravity scale 'felt' by the neutrinos. The above bounds indicate a value $M_{QG} \sim 10^{27}$ GeV for $E = \mathcal{O}(\text{few GeV})$ which are typical in atmospheric neutrino experiments. This is much higher than the theoretically expected (on naturalness grounds) Planck value $M_{QG} \sim 10^{19}$ GeV. This is to be contrasted with the case of kaons [2], where, as mentioned above, the bound on the decoherence parameter $\gamma < 10^{-21}$GeV obtained from CPLEAR [17], was found to be much closer to the Planck scale, the natural QG scale. On the other hand, the case (c), with an inverse energy E dependence, is purely phenomenological and does not stem directly from a specific model of decoherence. The bound on $\gamma_0 < 10^{-23}\,(\mathrm{GeV})^2$ obtained in that case is of the order of the quantity $\Delta m^2/E$ for atmospheric neutrino energies, and square mass differences of order $10^{-5}\,\mathrm{eV}^2$ which is assumed to be the right order for solar neutrino oscillations in the conventional oscillation scenario.

Unfortunately, the analysis of the data presented in [23] within a two-generation scenario did not show any concrete evidence for such a type-(c) decoherence, even after inclusion [27] of the first K2K data [28], in combination with Super Kamiokande (SK) data [29]. This prompted these authors to draw the pessimistic conclusion that the extension of the neutrino oscillation plus decoherence scenario to the fully-fledged case of three neutrino generations was not worthy. Adding to this pessimism, there were also some theoretical estimates of the decoherence parameter in *specific* models of quantum-gravity induced decoherence for generic two-level systems [30], according to which $\gamma_0 \sim (\Delta m^2)^2/E^2 M_P$, with Δm^2 the mass-squared difference between, say, the two neutrinos (or kaons etc). For $M_P \sim 10^{19}$ GeV, the Planck scale, this is beyond any prospects for experimental detection in oscillation experiments in the foreseeable future. However, as remarked in [22], such estimates are specific to the model considered in [30], and indeed there are concrete examples of theoretical (stringy) models of quantum-gravity-induced decoherence where they are not applicable. We shall discuss such issues later on in our article.

4. Extension to three generations: combining oscillations with decoherence

Pessimistic thoughts have also appeared in the work of [26], which considered the extension of the completely positive decoherence scenario to the standard three-generation neutrino oscillations case. The above-described three-state Lindblad problem has been adopted for the discussion of this case. The relativistic neutrino hamiltonian $H_{\rm eff} \sim p^2 + m^2/2p$, with m the neutrino mass, has been used as the hamiltonian of the subsystem in the evolution of eq. (2.3).

In terms of the generators $\mathcal{J}_\mu$, $\mu = 0, \ldots, 8$ of the SU(3) group, $H_{\rm eff}$ can be expanded as [26]: $\mathcal{H}_{\rm eff} = \frac{1}{2p}\sqrt{2/3}\left(6p^2 + \sum_{i=1}^3 m_i^2\right)\mathcal{J}_0 + \frac{1}{2p}(\Delta m_{12}^2)\mathcal{J}_3 + \frac{1}{2\sqrt{3}p}\left(\Delta m_{13}^2 + \Delta m_{23}^2\right)\mathcal{J}_8$, with the obvious notation $\Delta m_{ij}^2 = m_i^2 - m_j^2$, $i, j = 1, 2, 3$.

The analysis of [26] assumed *ad hoc* a diagonal form for the 9×9 decoherence matrix $\mathcal{L}$ in (2.4): $[\mathcal{L}_{\mu\nu}] = {\rm Diag}(0, -\gamma_1, -\gamma_2, -\gamma_3, -\gamma_4, -\gamma_5, -\gamma_6, -\gamma_7, -\gamma_8)$ in direct analogy with the two-level case of complete positivity [23, 25]. As we have mentioned already, there is no strong physical motivation behind such restricted forms of decoherence. This assumption, however, leads to the simplest possible decoherence models, and, for our *phenomenological* purposes in this work, we will assume the above form, which we will use to fit all the available neutrino data. It must be clear to the reader though, that such a simplification, if proven to be successful (which, as we shall argue below, is the case here), just adds more in favor of decoherence models, given the restricted number of available parameters for the fit in this case. In fact, any other non-minimal scenario will have it easier to accommodate data because it will have more degrees of freedom available for such a purpose.

Specifically we shall look at transition probabilities [26]:

$$P(\nu_\alpha \to \nu_\beta) = {\rm Tr}[\rho^\alpha(t)\rho^\beta] = \frac{1}{3} + \frac{1}{2}\sum_{i,k,j} e^{\lambda_k t}\mathcal{D}_{ik}\mathcal{D}_{kj}^{-1}\rho_j^\alpha(0)\rho_i^\beta \tag{4.1}$$

where $\alpha, \beta = e, \mu, \tau$ stand for the three neutrino flavors, and Latin indices run over $1, \ldots, 8$. The quantities λ_k are the eigenvalues of the matrix $\mathcal{M}$ appearing in the evolution (2.4), after taking into account probability conservation, which decouples $\rho_0(t) = \sqrt{2/3}$, leaving the remaining equations in the form: $\partial\rho_k/\partial t = \sum_j \mathcal{M}_{kj}\rho_j$. The matrices $\mathcal{D}_{ij}$ are the matrices that diagonalize $\mathcal{M}$ [21]. Explicit forms of these matrices, the eigenvalues λ_k, and consequently the transition probabilities (4.1), are given in [26].

The important point to stress is that, in generic models of oscillation plus decoherence, the eigenvalues λ_k depend on both the decoherence parameters γ_i and the mass differences Δm_{ij}^2. For instance, $\lambda_1 = \frac{1}{2}[-(\gamma_1 + \gamma_2) - \sqrt{(\gamma_2 - \gamma_1)^2 - 4\Delta_{12}^2}]$, with the notation $\Delta_{ij} \equiv \Delta m_{ij}^2/2p$, $i, j = 1, 2, 3$. Note that, to leading order in the (small) squared-mass differences, one may replace p by the total neutrino energy E, and this will be understood in what follows. For detailed expressions of the rest of the parameters we refer the reader to [26]. However, we note, for future use, that it is a generic feature of the λ_k to depend on the quantities Ω_{ij} which are given by

$$\begin{aligned}
\Omega_{12} &= \sqrt{(\gamma_2 - \gamma_1)^2 - 4\Delta_{12}^2} \\
\Omega_{13} &= \sqrt{(\gamma_5 - \gamma_4)^2 - 4\Delta_{13}^2} \\
\Omega_{23} &= \sqrt{(\gamma_7 - \gamma_6)^2 - 4\Delta_{23}^2}\,.
\end{aligned} \tag{4.2}$$

From the above expressions for the eigenvalues λ_k, it becomes clear that, when decoherence and oscillations are present simultaneously, one should distinguish two cases, according to the relative magnitudes of Δ_{ij} and $\Delta\gamma_{kl} \equiv \gamma_k - \gamma_l$: (i) $2|\Delta_{ij}| \geq |\Delta\gamma_{k\ell}|$, and (ii) $2|\Delta_{ij}| < |\Delta\gamma_{k\ell}|$. In the former case, the probabilities (4.1) contain trigonometric (sine and cosine) functions, whilst in the latter they exhibit hyperbolic sin and cosine dependence.

Assuming mixing between the flavors, amounts to expressing neutrino flavor eigenstates $|\nu_\alpha >$, $\alpha = e, \mu, \tau$ in terms of mass eigenstates $|\nu_i >$, $i = 1, 2, 3$ through a (unitary) matrix U: $|\nu_\alpha >= \sum_{i=1}^{3} U^*_{\alpha i}|\nu_i >$. This implies that the density matrix of a flavor state ρ^α can be expressed in terms of mass eigenstates as: $\rho^\alpha = |\nu_\alpha >< \nu_\alpha| = \sum_{i,j} U^*_{\alpha i} U_{\alpha j} |\nu_i >< \nu_j|$. From this we can determine $\rho^\alpha_\mu = 2\mathrm{Tr}(\rho^\alpha \mathcal{J}_\mu)$, a quantity needed to calculate the transition probabilities (4.1).

The important comment we would like to raise at this point is that, when considering the above probabilities in the antineutrino sector, the respective decoherence parameters $\bar{\gamma}_i$ in general may be different from the corresponding ones in the neutrino sector, as a result of the strong form of CPT violation. In fact, as we shall discuss next, this will be crucial for accommodating the LSND result without conflicting with the rest of the available neutrino data. This feature is totally unrelated to mass differences between flavors.

5. Combining decoherence, oscillations and the LSND result

In [26] a pessimistic conclusion was drawn on the "clear incompatibility between neutrino data and theoretical expectations", as followed by their qualitative tests for decoherence. It is a key feature of our present work to point out that we do not share at all this point of view. In fact, as we shall demonstrate below, if one takes into account all the available neutrino data, including the final LSND results [7], which the authors of [26] did not do, and allows for the above mentioned CPT violation in the decoherence sector, then one will arrive at exactly the opposite conclusion, namely that three-generation decoherence and oscillations can fit the data successfully!

As we shall argue here, compatibility of all available data, including CHOOZ [31] and LSND, can be achieved through a set of decoherence parameters γ_j with energy dependences $\gamma_j^0 E$ and γ_j^0/E, with $\gamma_j^0 \sim 10^{-18}, 10^{-24}\,(\mathrm{GeV})^2$, respectively, for some j's. As it will be clear later, the quality of the fit diminishes to the level of the standard three-oscillation case once KamLAND spectral distortion data [32] are included but is significanlty higher otherwise, showing that either additional energy dependences are needed or non-diagonal elements are required to get better agreement with data.

Some important remarks are in order. First of all, in the analysis of [26] pure decoherence is excluded in three-generation scenaria, as in two generation ones, due to the fact that the transition probabilities in the case $\Delta m_{ij}^2 = 0$ (pure decoherence) are such that the survival probabilities in both sectors are equal, i.e. $P(\nu_\alpha \to \nu_\alpha) = P(\overline{\nu}_\alpha \to \overline{\nu}_\alpha)$. From (4.1) we have in this case [26]:

$$P_{\nu_e \to \nu_e} = P_{\nu_\mu \to \nu_\mu} \simeq \frac{1}{3} + \frac{1}{2} e^{-\gamma_3 t} + \frac{1}{6} e^{-\gamma_8 t} \,. \tag{5.1}$$

From the CHOOZ experiment [31], for which $L/E \sim 10^3/3\,\mathrm{m/MeV}$, we have that $\langle P_{\bar{\nu}_e \to \bar{\nu}_e} \rangle \simeq 1$, while the K2K experiment [28] with $L/E \sim 250/1.3\,\mathrm{km/GeV}$ has observed events compatible with $\langle P_{\nu_\mu \to \nu_\mu} \rangle \simeq 0.7$, thereby contradicting the theoretical predictions (5.1) of pure decoherence.

However, this conclusion is based on the fact that in the antineutrino sector the decoherence matrix is the same as that in the neutrino sector. In general this need not be the case, in view of CPT violation, which could imply a different interaction of the antiparticle with the gravitational environment as compared with the particle.

In our tests we take into account this possibility, but as can be seen from the figure, and will be evident later on, once the analysis will be explained in some depth, pure decoherence can be excluded also in this case, as it is clearly incompatible with the totality of the available data.

In order to check our model, we have performed a χ^2 comparison (as opposed to a χ^2 fit) to SuperKamiokande sub-GeV and multi GeV data (the forty data points that are shown in the figures), CHOOZ data (15 data points) LSND (1 datum) and KamLAND spectral information [32] (sampled in 13 bins), for a sample point in the vast parameter space of our extremely simplified version of decoherence models. Let us emphasize that we have **not** performed a χ^2-fit and therefore the point we are selecting (by "eye" and not by χ) is not optimized to give the best fit to the existing data. Instead, it must be regarded as one among the many equally good sons in this family of solutions, being extremely possible to find a better fitting one through a complete (and highly time consuming) scan over the whole parameter space.

Cutting the long story short, and to make the analysis easier, we have set all the γ_i in the neutrino sector to zero, restricting this way, all the decoherence effects to the antineutrino one where, we have assumed for the sake of simplicity,

$$\bar{\gamma}_i = \bar{\gamma}_{i+1} \qquad \text{for } i = 1,4,6,7 \qquad \text{and} \qquad \bar{\gamma}_1 = \bar{\gamma}_4 \,, \qquad \bar{\gamma}_3 = \bar{\gamma}_8 \,. \tag{5.2}$$

Furthermore, we have also set the CP violating phase of the NMS matrix to zero, so that all the mixing matrix elements become real.

With these assumptions, the otherwise cumbersome expression for the transition probability for the antineutrino sector takes the form,

$$P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta} = \frac{1}{3} + \frac{1}{2} \Bigg\{ \rho_1^\alpha \rho_1^\beta \cos\left(\frac{|\Omega_{12}|t}{2}\right) e^{-\bar{\gamma}_1 t} + \rho_4^\alpha \rho_4^\beta \cos\left(\frac{|\Omega_{13}|t}{2}\right) e^{-\bar{\gamma}_4 t} +$$

$$+ \rho_6^\alpha \rho_6^\beta \cos\left(\frac{|\Omega_{23}|t}{2}\right) e^{-\bar{\gamma}_6 t} + e^{-\bar{\gamma}_3 t}\left(\rho_3^\alpha \rho_3^\beta + \rho_8^\alpha \rho_8^\beta\right)\Bigg\} . \tag{5.3}$$

where the Ω_{ij} were defined in the previous section and are the same in both sectors (due to our choice of γ_i's) and

$$\begin{aligned}
\rho_1^\alpha &= 2\,\mathrm{Re}(U_{\alpha 1}^* U_{\alpha 2}) \\
\rho_3^\alpha &= |U_{\alpha 1}|^2 - |U_{\alpha 2}|^2 \\
\rho_4^\alpha &= 2\,\mathrm{Re}(U_{\alpha 1}^* U_{\alpha 3}) \\
\rho_6^\alpha &= 2\,\mathrm{Re}(U_{\alpha 2}^* U_{\alpha 3}) \\
\rho_8^\alpha &= \frac{1}{\sqrt{3}}(|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2 - 2|U_{\alpha 3}|^2)
\end{aligned} \tag{5.4}$$

where the mixing matrices are the same as in the neutrino sector. For the neutrino sector, as there are no decoherence effects, the standard expression for the transition probability is valid.

It is obvious now that, since the neutrino sector does not suffer from decoherence, there is no need to include the solar data into the fit. We are guaranteed to have an excellent agreement with solar data, as long as we keep the relevant mass difference and mixing angle within the LMA region, something which we shall certainly do.

As mentioned previously, CPT violation is driven by, and restricted to, the decoherence parameters, and hence masses and mixing angles are the same in both sectors, and selected to be

$$\begin{aligned}
\Delta m_{12}^2 &= \Delta\overline{m_{12}}^2 = 7 \cdot 10^{-5}\,\mathrm{eV}^2\,, \\
\Delta m_{23}^2 &= \Delta\overline{m_{23}}^2 = 2.5 \cdot 10^{-3}\,\mathrm{eV}^2\,, \\
\theta_{23} &= \overline{\theta_{23}} = \frac{\pi}{4}\,, \qquad \theta_{12} = \overline{\theta_{12}} = .45\,, \\
\theta_{13} &= \overline{\theta_{13}} = .05\,,
\end{aligned}$$

as indicated by the state of the art analysis.

For the decoherence parameters we have chosen (c.f. (5.2))

$$\overline{\gamma_1} = \overline{\gamma_2} = \overline{\gamma_4} = \overline{\gamma_5} = 2 \cdot 10^{-18} \cdot E$$

and

$$\overline{\gamma_3} = \overline{\gamma_6} = \overline{\gamma_7} = \overline{\gamma_8} = \frac{1 \cdot 10^{-24}}{E}\,, \tag{5.5}$$

where E is the neutrino energy, and barred quantities refer to the antineutrinos, given that decoherence takes place only in this sector in our model. All the other parameters are assumed to be zero. All in all, we have introduced only two new parameters, two new degrees of freedom, $\overline{\gamma_1}$ and $\overline{\gamma_3}$, and we shall try to explain with them all the available experimental data.

It can be checked straightforwardly, by means of (5.3), that for the regime of the parameters relevant to the experiments we are considering in this work, this parametrization guarantees the positivity of the relevant probabilities. This is an important issue, because usually negative probabilities are viewed as a signal of inconsistency in parameterizing the pertinent decoherence effects. In our case, with two different energy dependences in the (non constant) decoherence coefficients, there seems to be no master condition that guarantees the complete positivity in a way independent of the energies/momenta of the neutrinos. As we shall discuss later on in the article, we attribute the $1/E$-dependent decoherent coefficients to conventional matter effects, while the E-dependent ones are associated with novel effects of quantum gravity, which increase with the energy of the probe. The negative probabilities that may occur outside the regime of parameters to be specified below are then interpreted as implying simply that our linear (and simplified) parametrization of quantum gravity decoherence (5.5) ceases to be valid in such regimes, and more complicated, probably non linear, entanglement may be in operation, as expected in a quantum theory of gravity [22].

With these in mind, we now observe that, in our simplified model of decoherence, parametrized by a diagonal decoherence matrix, positivity of the relevant probabilities seems to be guaranteed for $\overline{\gamma}_1 L > \overline{\gamma}_3 L$, which, with our parametrization (5.5), implies: $L/E \leq 10^{24} - 10^{25}$ GeV^{-2}, and $E > 1\,\mathrm{MeV}$. These sufficient conditions are met by all the current and planned terrestrial (anti)neutrino experiments. Outside this regime, our parametrization simply fails, and one needs to resort to more complicated situations, which fall beyond the scope of the present paper, and will be presented elsewhere.

At this point it is important to stress that the inclusion of two new degrees of freedom is not sufficient to guarantee that one will indeed be able to account for all the experimental observations. We have to keep in mind that, in no-decoherence situations, the addition of a sterile neutrino (which comes along with four new degrees of freedom -excluding again the possibility of CP violating phases) did not seem to be sufficient for matching all the available experimental data, at least in CPT conserving situations.

In order to test our model with these two decoherence parameters in the antineutrino sector, we have calculated the zenith angle dependence of the ratio "observed/(expected in the no oscillation case)", for muon and electron atmospheric neutrinos, for the sub-GeV and multi-GeV energy ranges, when mixing is taken into account. Since matter effects are important for atmospheric neutrinos, we have implemented them through a two-shell model, where the density in the mantle (core) is taken to be roughly 3.35 (8.44) gr/cm^3, and the core radius is taken to be 2887 km. We should note at this stage that a "fake" CPT Violation appears due to matter effects, arising from a relative sign difference of the matter potential between the respective interactions of neutrinos and antineutrinos with ordinary matter. This, however, is easily disentangled from our genuine (due to quantum gravity) CPT Violation, used here to parametrize our model fit to LSND results; indeed, a systematic study of such effects [33] has shown that "fake" CPT Violation increases with the oscillation length, but decreases with the neutrino energy, E, vanishing in the limit $E \to \infty$; moreover, no independent information regarding such effects can be obtained by looking at the antineutrino sector, as compared with data from the neutrino sector, due to the fact that in the presence of "fake" CPT Violation, but in the absence of any genuine CPT breaking, the pertinent CPT probability differences between neutrinos and

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antineutrinos are related, $\Delta P^{\rm CPT}_{\alpha\beta} = -\Delta P^{\rm CPT}_{\overline{\beta}\overline{\alpha}}$, where $\Delta P^{\rm CPT}_{\alpha\beta} = P_{\alpha\beta} - P_{\overline{\beta}\overline{\alpha}}$, and the Greek indices denote neutrino flavors. These features are to be contrasted with our dominant decoherence effects $\overline{\gamma}_1$ (5.5), proportional to the antineutrino energy, E, which are dominant only in the antineutrino sector. For the same reason, our effects can be disentangled from "fake" decoherence effects arising from gaussian averages of the oscillation probability due to, say, uncertainties in the energy of the neutrino beams [34], which are the same for both neutrinos and antineutrinos. We, therefore, claim that the complex energy dependence in (5.5), with *both* $L \cdot E$ and L/E terms being present in the antineutrino sector, may be a characteristic feature of new physics, with the $L \cdot E$ terms being related to quantum-gravity induced (genuine) CPT Violating decoherence.

The results are shown in figure 1c, where, for the sake of comparison, we have also included the experimental data. We also present in that figure the pure decoherence scenario in the antineutrino sector (a), as well as in both sectors (b). For completeness, we also present a scenario with neutrino mixing but with decoherence operative in both sectors (d). The conclusion is straightforward: while pure decoherence appears to be excluded,decoherence plus mixing provides an astonishing agreement with experiment.[1]

As bare eye comparisons can be misleading, we have also calculated the χ^2 value for each of the cases, defining the atmospheric χ^2 as

$$\chi^2_{\rm atm} = \sum_{M,S} \sum_{\alpha=e,\mu} \sum_{i=1}^{10} \frac{(R^{\rm exp}_{\alpha,i} - R^{\rm th}_{\alpha,i})^2}{\sigma^2_{\alpha i}} . \tag{5.6}$$

Here $\sigma_{\alpha,i}$ are the statistical errors, the ratios $R_{\alpha,i}$ between the observed and predicted signal can be written as

$$R^{\rm exp}_{\alpha,i} = \frac{N^{\rm exp}_{\alpha,i}}{N^{\rm MC}_{\alpha,i}} \tag{5.7}$$

(with α indicating the lepton flavor and i counting the different bins, ten in total) and M, S stand for the multi-GeV and sub-GeV data respectively. For the CHOOZ experiment we used the 15 data points with their statistical errors, where in each bin we averaged the probability over energy and for LSND one datum has been included. The results with which we hope all our claims become crystal clear are summarized in table 1 and 2, were we present the χ^2 comparison for the following cases: (a) pure decoherence in the antineutrino sector, (b) pure decoherence in both sectors, (c) mixing plus decoherence in the antineutrino sector, (d) mixing plus decoherence in both sectors, and (e) mixing only - the standard scenario for all experimental data with (table 2) and without (table 1) KamLAND spectral distortion.

From the tables it becomes clear that the mixing plus decoherence scenario in the antineutrino sector can easily account for all the available experimental information, including LSND when KamLAND spectral distortion data are excluded but does not significantly improve over the standard case once they are included, showing clearly the limitations of our simplified scenario.

[1]Pure decoherence models, are not ruled out by the present analysis, which tests only one particular point in a huge parameter space, but by a complete scan like the ones in [26, 27].

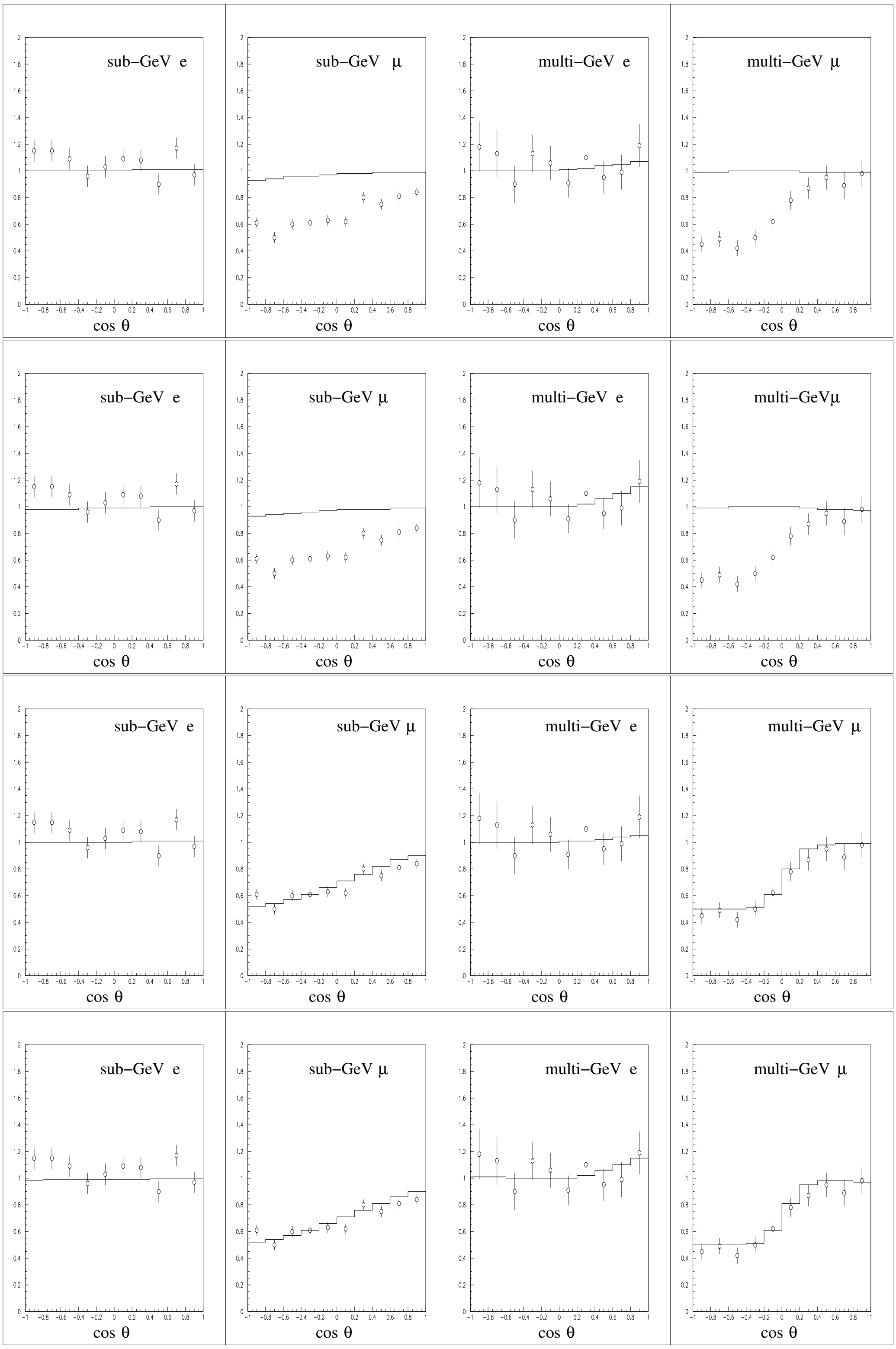

Figure 1: Decoherence fits, from top to bottom: (a) pure decoherence in antineutrino sector, (b) pure decoherence in both sectors, (c) mixing plus decoherence in the antineutrino sector only, (d) mixing plus decoherence in both sectors. The dots correspond to SK data.

model	χ^2 without LSND	χ^2 including LSND
(a)	1097.6	1104.3
(b)	1037.8	1044.4
(c)	45.7	45.9
(d)	52.5	52.7
(e)	53.9	60.7

Table 1: χ^2 obtained for (a) pure decoherence in antineutrino sector, (b) pure decoherence in both sectors, (c) mixing plus decoherence in the antineutrino sector only, (d) mixing plus decoherence in both sectors, (e) standard scenario with and without the LSND result for all experimental data but KamLAND spectral distortions.

model	χ^2 without LSND	χ^2 including LSND
(a)	1135.8	1142.3
(b)	1095.6	1102.3
(c)	83.9	84.2
(d)	90.7	90.9
(e)	78.6	85.4

Table 2: χ^2 obtained for (a) pure decoherence in antineutrino sector, (b) pure decoherence in both sectors, (c) mixing plus decoherence in the antineutrino sector only, (d) mixing plus decoherence in both sectors, (e) standard scenario with and without the LSND result, for all data.

Such a change can be easily undestood by noticing that scenario (c) improves the fit over the standard case (case (e)) by $\Delta\chi^2 = 14.8$ (see table 1) but the price to pay for it is to give up spectral distortions in KamLAND. In fact, for KamLAND data alone we find a $\chi^2 = 24.7$ for case (e) and $\chi^2 = 38.4$ for an energy independent suppression (case (c)), implying a $\Delta\chi^2 = 13.7$, which essentialy means that the net effect is to put our decoherence model back to the standad case level.

It is important to stress once more that our sample point was not obtained through a scan over all the parameter space, but by an educated guess, and therefore plenty of room is left for improvements. On the other hand, for the mixing-only/no-decoherence scenario, we have taken the best fit values of the state of the art analysis and therefore no significant improvements are expected. At this point a word of warning is in order: although superficially it seems that scenario (d), decoherence plus mixing in both sectors, provides an equally good fit, one should remember that including decoherence effects in the neutrino sector can have undesirable effects in solar neutrinos, especially due to the fact that decoherence effects are weighted by the distance traveled by the neutrino, something that may lead to seizable (not observed!) effects in the solar case.

Another point to stress is that, as can be easily seen in the figure, decoherence plays no role for atmospheric neutrino energies and baselines, where the effect is given almost totally by the oscillations driven by the mixing. This was somehow expected, since decoherence is known not to be the leading force behind the atmospheric neutrinos. In fact we are taking

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advantage of the fact that for our particular set of parameters, no significant effects are expected in the (2,3) channels.

One might wonder then, whether decohering effects, which affect the antineutrino sector sufficiently to account for the LSND result, have any impact on the solar-neutrino related parameters, measured through antineutrinos in the KamLAND experiment [35, 32]. In order to answer this question, it will be sufficient to calculate the electron survival probability for KamLAND in our model, which turns out to be $P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta} \mid_{\rm KamLAND} \simeq .57$, in perfect agreement with observations. As is well known, KamLAND is sensitive to a bunch of different reactors with distances spanning from 80 to 800 km. However, the bulk of the signal comes from just two of those, whose distances are 160 and 179 km. These parameters have been used to compute the survival probability. Nevertheless, decohering effects, at least of the simplified type discussed in this work, are not able to account for any spectral distortion; they rather provide an overall suppression. Thus, the spectral distortion that is apparently being observed by KamLAND [32], tends to favor the standard three-generation scenario (without KamLAND spectral information but including LSND the χ^2 for models (c) and (e) would be 45.9 and 60.7, respectively). Therefore, if KamLAND evidence gets stronger a simplified decoherence model of the class discussed here would be ruled-out.However, this should by no means be regarded as implying that decoherence models in general will not survive such a case. On the contrary, it is our belief that CPT Violating decoherence models with complex energy dependences of the decoherence parameters as the one discussed here, but probably of more complicated, even non linear form, stand a good chance of describing nature.

It is also interesting to notice that in our model, the LSND effect is not given by the phase inside the oscillation term (which is proportional to the mass difference) but rather by the decoherence factor multiplying the oscillation term. Therefore the tension between LSND and KARMEN [36] data is naturally eliminated, because the difference in length leads to an exponential suppression. Thus, while we predict a 0.24 % anti-electron neutrino appearance probability for LSND, the corresponding one for KARMEN gets only to 0.14%. And although this number is already below their experimental sensitivity, one should notice that KARMEN combines neutrino and antineutrino channels in their analysis, so that the actual appearance probability is even smaller, as our effect shows up only in the antineutrino sector.

Another potential source of concern for the present model of decoherence might be accelerator neutrino experiments, which involve high energies and long baselines, and where the decoherence $L \cdot E$ scaling can potentially be probed. This, however, is not the case. Accelerator experiments typically join their neutrino and antineutrino data, with the antineutrino statistics being always smaller than the neutrino one. This fact, together with the smaller antineutrino cross section, renders our potential signal consistent with the background contamination. Even more, in order to constrain decoherence effects of the kind we are proposing here through accelerator experiments, excellent control and knowledge of the beam background are mandatory. The new KTeV data [37] on kaon decay branching ratios, for example, will change the ν_e background enough to make any conclusion on the viability of decoherence models useless. After all, the predicted signal in our decoherence scenario will be at the level of the electron neutrino contamination, and therefore one would need to disentangle one from the other.

Having said that, it is clear that althougth this very simplified decoherence model (once neutrino mixing is taken into account) does not account for all the observations including the LSND result better than the standard three-neutrino model, it certainly suggests that less simplified models are worth exploring. This scenario, which makes dramatic predictions for the upcoming neutrino experiments, expresses a strong observable form of CPT violation in the laboratory, and, in this sense, our fit does not seem to give (yet) a conclusive answer to the question asked in the introduction as to whether the weak form of CPT invariance (2.1) is violated in Nature, but it certainly encourages further studies.

This CPT violating pattern, with equal mass spectra for neutrinos and antineutrinos, will have dramatic signatures in future neutrino oscillation experiments. The most striking consequence will be seen in MiniBooNE [38], According to our picture, MiniBooNE will be able to confirm LSND only when running in the antineutrino mode and not in the neutrino one, as decoherence effects live only in the former. Smaller but experimentally accessible signatures will be seen also in MINOS [39], by comparing conjugated channels (most noticeably, the muon survival probability).

We next remark that fits with decoherence parameters with energy dependences of the form (5.5) imply that the exponential factors $e^{\lambda_k t}$ in (4.1) due to decoherence will modify the amplitudes of the oscillatory terms due to mass differences, and while one term depends on L/E the other one is driven by $L \cdot E$, where we have set $t = L$, with L the oscillation length (we are working with natural units where $c = 1$).

The order of the coefficients of these quantities, $\gamma_j^0 \sim 10^{-18}, 10^{-24}\,(\mathrm{GeV})^2$, found in our sample point, implies that for energies of a few GeV, which are typical of the pertinent experiments, such values are not far from $\gamma_j^0 \sim \Delta m_{ij}^2$. If our conclusions survive the next round of experiments, and therefore if MiniBOONE experiment [38] confirms previous LSND claims, then this may be a significant result. One would be tempted to conclude that if the above estimate holds, this would probably mean that the neutrino mass differences might be due to quantum gravity decoherence. Theoretically it is still unknown how the neutrinos acquire a mass, or what kind of mass (Majorana or Dirac) they possess. There are scenaria in which the mass of neutrino may be due to some peculiar backgrounds of string theory for instance. If the above model turns out to be right we might then have, for the first time in low energy physics, an indication of a direct detection of a quantum gravity effect, which disguised itself as an induced decohering neutrino mass difference. Notice that in our sample point only antineutrinos have non-trivial decoherence parameters $\overline{\gamma_i}$, for $i = 1$ and 3, while the corresponding quantities in the neutrino sector vanish. This implies that there is a single cause for mass differences, the decoherence in antineutrino sector, which is compatible with common mass differences in both sectors. If it turns out to be true, this would be truly amazing.

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6. Speculations on theoretical models of decoherence

Moreover, if the neutrino masses are actually related to decoherence as a result of quantum gravity, this may have far reaching consequences for our understanding of the Early stages of our Universe, and even the issue of Dark Energy that came up recently as a result of astrophysical observations on a current acceleration of the Universe from either distant supernovae data [40] or measurements on Cosmic Microwave Background temperature fluctuations from the WMAP satellite [41]. Indeed, as discussed in [22, 4], decoherence implies an absence of a well-defined scattering S-matrix, which in turn would imply CPT violation in the strong form, according to the theorem of [14]. A positive cosmological *constant* $\Lambda > 0$ will also lead to an ill definition of an S-matrix, precisely due to the existence, in such a case, of an asymptotic-future de Sitter (inflationary) phase of the universe, with Hubble parameter $\sim \sqrt{\Lambda}$, implying the existence of a cosmic (Hubble) horizon. This in turn will prevent a proper definition of pure asymptotic states.

We would like to point out at this stage that the claimed value of the dark energy density component of the (four-dimensional) Universe today, $\Lambda \sim 10^{-122} M_P^4$, with $M_P \sim 10^{19}$ GeV (the Planck mass scale), can actually be accounted for (in an amusing coincidence?) by the scale of the neutrino mass differences used in order to explain the oscillation experiments. Indeed, $\Lambda \sim [(\Delta m^2)^2/M_P^4] M_P^4 \sim 10^{-122} M_P^4$ for $\Delta m^2 \sim 10^{-5}\,\mathrm{eV}^2$, the order of magnitude of the solar neutrino mass difference assumed in oscillation experiments (which is the one that encompasses the decoherence effects, as can be seen by eq. (5.3)). The quantum decoherence origin of this mass then would be in perfect agreement with the decoherence properties of the cosmological constant vacuum, mentioned previously. We note at this stage that it is possible to justify theoretically the above form for $\Lambda \propto (\Delta m^2)^2$, if one computes the vacuum expectation value of the stress tensor of the neutrino field in a Robertson-Walker space-time background, not with respect to the mass-eigenstate vacuum, but with respect to a "flavour vacuum", which may not be unitarily equivalent to the former. Details are given in [42] and references therein, where we refer the interested reader. We only note here that the issue is related to the old problem of mixing in a quantum field theory, which has been studied in detail in [43]. Needless to say that we by no means consider such a problem as completely understood to date, but we would simply like to point out here, and in [42], yet another interesting feature, namely a possible link of the neutrino mass difference to a non zero, non perturbative, contribution to the cosmological constant.

Unfortunately, at present we have no concrete theoretical model to explain the above-described decoherence in the antineutrino sector, with the non universal energy dependence (5.5). Conventional quantum-gravity decoherence scenaria assume usually a universal CPT conserving character for the decoherence parameters between the particle and antiparticle sectors, and in fact they yield too small decoherence parameters to be detected experimentally. For instance, in the two-level double-commutator model of quantum-gravity induced decoherence of [30], the double commutator structure in the decoherence part yields the estimate: $\gamma \sim (\Delta E)^2/M_P$ for a four-dimensional quantum-gravity decoherence model. Such an estimate stems from the double commutator of the Lindblad operators

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$[b_i, b_i, \rho]$ when considered between appropriate energy eigenstates. Here, ΔE is the energy variance, which in the case of neutrinos may be estimated from the hamiltonian $p + m^2/2p$ to be $\Delta E \sim (\Delta m^2)/E$, thereby yielding a decoherence model with $\gamma \propto (\Delta m^2)^2/M_P E^2$, which is universal between particle and antiparticle sectors, and actually too small to be detected in oscillation experiments.

It is possible, however, that other models yield significantly larger values to match our estimates for the decoherence parameters above. For instance, let us consider a crude model of decoherence, based on non-critical string theory [22], according to which a modified Lindblad evolution of the type (2.3) can be derived as a result of departure from world-sheet conformal invariance. The latter describes the interaction of string matter with 'fuzzy' space-time singular backgrounds, entailing 'loss of information'. Using for the moment some technical jargon, we mention that departure from conformal invariance is formally manifested through the appearance of a central charge deficit in the corresponding σ model [22], which from a target-space viewpoint may resemble a cosmological constant Λ. Universes with a cosmological constant are not conformal backgrounds of string theory, as a result of the asymptotic future Hubble horizon. In σ-models describing the interaction of matter with foamy backgrounds such Λ does not have to be as small as the actual cosmological constant of our Universe, mentioned above. In fact it could be due to string loop corrections, that is higher world-sheet topologies, and this may lead to violations of conformal invariance of order of the string scale M_s.

It has been argued in [11] that such models of non critical string theory lead to a Lindblad type modified evolution for reduced density matrices of four-dimensional string matter of the form:

$$\partial_t \rho = i[\rho, H_{\text{eff}}] + \mathcal{L} : g_{\mu\nu}[g^{\mu\nu}, \rho] : , \qquad \mathcal{L} = \mathcal{O}(M_s \Lambda) \tag{6.1}$$

where M_s is the string scale, which, in general, may be different from the four-dimensional Planck mass $M_P \sim 10^{19}$ GeV, $g^{\mu\nu}$ is the (fuzzy) metric background in ten dimensions (the critical dimension of string theory), the: $\ldots$: indicate quantum ordering of the appropriate operators, and Λ denotes now the ten-dimensional dimensionless central-charge deficit, which, if due to string loop corrections, it could be of order one (in natural string (M_s) units). One may select an antisymmetric quantum ordering prescription, which leads to a double commutator structure of the decoherence terms $\mathcal{L} : [g_{\mu\nu}, [g^{\mu\nu}, \rho]] :$. This will be a local contribution to matter propagation, due to interactions of matter with space time foam, and thus will be dominant as compared to global contributions due to a non-zero Dark energy component of the Universe [11]. It remains a challenge of course to explain how such a local decoherence will not lead to large global contributions to the vacuum energy.

The decoherence terms in (6.1) are essentially proportional to the world-sheet β function for the graviton in a 'cosmological constant' space time with metric $g_{\mu\nu}$, i.e. of the form $\beta_{\mu\nu} = M_s^{-2} R_{\mu\nu} \sim \Lambda g_{\mu\nu}$ [11], where $R_{\mu\nu}$ is the Ricci tensor of the cosmological constant ten-dimensional non-conformal string background (an $\mathcal{O}(\alpha' = M_s^{-2})$ analysis for the world-sheet β-functions of the pertinent σ-model suffices for our purposes).

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If we accept this scenario for time evolution of matter (or antimatter), then, in our case of a non-zero string-loop induced cosmological constant, upon projecting on string theory space states $|g^i>$, where g^i are background fields in the target space over which the matter string propagates (such as $g_{\mu\nu}$ etc.), the double commutator structure yields simply [22] the square of the variance of the metric background as a result of interaction with stringy matter $(\Delta g_{\mu\nu})^2$. This expresses essentially the back reaction of matter onto space time, as a result of the quantum properties of the foam.

The form of this variance depends on the details of the model, and it is conceivable, although we have no concrete model in mind at this stage, that the variances are non zero only for the antineutrino sector, and such that they allow to write (6.1) in the form (2.4), where the Lindblad operator is diagonal, with various entries that could match our estimates above. In the case of antineutrino decoherence, for instance, one may associate the variance with the uncertainty in the measurement of a typical length in the problem, which is the oscillation length. In our model the latter is of order $L \sim E/\Delta m^2$ in both particle and antiparticle sectors. Thus, an uncertainty in $\Delta g_{\mu\nu}$ may be associated with an uncertainty in the covariant length [44] $\Delta L^2 \sim L^2 \Delta g_{\mu\nu}$, from which we obtain the estimate: $(\Delta g_{\mu\nu})^2 \sim (\Delta L^2/L^2)^2 \sim (\delta E/E)^2$. Here, δE denotes an antineutrino, say, energy uncertainty due to the 'fuzziness' of space time, and should not be confused with the energy variance between neutrino states mentioned above in the context of the model of [30].

It is such uncertainties that could be energy dependent in a peculiar way only for the antineutrino sector, as a result of the strong form of CPT violation due to decoherence [14], e.g. $\delta E \sim E(E/M_s)^\alpha$, for some α which are in general different for the different decoherence parameters, and yield the complicated energy dependence argued in our work above. In fact, if the conformal invariance violations in (6.1) are large only in the antineutrino sector, then this may provide the natural explanation as to why the dark energy component of the Universe is still small, given that the latter will be due to matter, and there is a huge matter-antimatter asymmetry in the Universe. At present, however, we have no answers to such issues, and our remarks above should be treated with caution. We find it challenging, nevertheless, to try and interpret theoretically, in terms of microscopic models, the complicated energy dependence of our decoherence parameters (5.5). This is currently under progress.

In any case, given that there is no full/complete theory of decoherence, any guess on the order of magnitude of the coefficients may be proven wrong. This fact encourages phenomenological analyses like ours, whose results can be contrasted to experimental data and may remain valid independently of the theoretical speculations on the origin and size of the γ's.

7. Instead of conclusions

In this work we have presented a set of decoherence parameters which can account for all available neutrino data, including LSND results. By invoking a three-generation decoherence scenario, combined with conventional oscillations, and flavor violations, we have managed to fit the available data and to obtain decoherence parameters which could find

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a natural explanation in some theoretical models of quantum gravity. We explained the LSND results not by assuming CPT violating mass differences, but by invoking CPT violation in its strong form, namely that the decoherence parameters differ between neutrinos and antineutrinos. The agreement of this fit with the data was found to be better than in the two-generation decoherence case [23], where there was no evidence for decoherence. This conclusion depends crucially upon the inclusion of the LSND results. Our analysis and conclusions are therefore in disagreement with pessimistic claims made in the recent literature [27, 26] on the non-necessity of studies of quantum-gravity-induced decoherence in three generation models.

It is imperative that the LSND results are confirmed by future experiments, such as MiniBOONE [38], because as we have seen above, it is indeed possible that the study of neutrino physics constitutes a true and rare window to Planck scale physics, with far reaching consequences ranging from cosmological considerations to the origin of the dark energy component of the Universe. In the present work, motivated by our sample point, we put forward a conjecture that quantum-gravity decoherence may be responsible for the neutrino mass differences and the non-zero value of the vacuum energy density of the Universe today. The scaling of the decoherence parameter with the energy is a crucial test of such conjectures, and would be a useful theoretical guide for model building, should the results of LSND and the present work survive future data.

Before closing we must stress the fact that the dynamical semi-groups approach to decoherence, examined here, relies on the approximation that the quantum gravity effects imply a linear decoherence, i.e. a linear dependence on the reduced density matrix of matter. This may not be true in a complete theory of quantum gravity, and hence, even if we get agreement with experimental data, this agreement should be interpreted with great care. Clearly we are in need of a complete and detailed mathematical model of quantum gravity, because generic and model independent analyses may be misleading. Nevertheless, the possibility of verifying experimentally deviations from conventional field theoretical treatments in the near future, e.g. in the context of neutrino physics, is by itself exciting even if it cannot give a complete answer to such fundamental questions as the Planck scale structure of quantum space time.

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